BULLETIN OF THE
GREEK MATHEMATICAL SOCIETY
Volume 53, 2007 (119–129)

Quotient Local Uniformities

Errikos Papatriantafillou

Received 27/11/2004 Accepted 06/04/2006

0. Introduction

Quotient structures in the theory of uniform spaces have been considered by R.W. Bagley [1] and C.J. Himmelberg [5](see also [14]). Moreover, W. Carlson [2] studied quotient structures in quasi-uniform spaces. The more general case of semi-uniform spaces [8] in connection with quotient structures was already considered in [9], [10] (cf. also [12]).

Our aim in this paper is to study quotient structures in the case of *locally uniform spaces*; these spaces were considered by J. Williams [15] and also, independently, by this author in [8]. Furthermore, the same notion was employed by F. Jeschek [4] and also in [12], in connection with the classical Ascoli-Arzela Theorem.

More precisely, the content of the paper is as follows:If $X[Q]$ is a locally uniform space and R an equivalence relation on X, let $p : X \to X/R$ be the (canonical) quotient map. So we give in Section 1 necessary and sufficient conditions such that the "direct image" of Q, $(p\times p)(Q)$, be a local uniformity, so that it is then a quotient local uniformity, as well (see Theorem 1.1 and Remark 1.1, iii)). This (quotient) local uniformity is also the image via p of the largest saturated local uniformity contained in Q (same theorem, in connection with Proposition 1.1). In section 2, taking a locally uniform space $X[Q]$ with X equipped with the induced topology from Q, we give sufficient conditions such that the quotient local uniformity defines the quotient topology on X/R (see, for instance, Theorem 2.3).

We continue in Section 3 by considering the equivalence relation I_Q in X determined by the intersection of the members of Q the so-called "atom" of Q, a notion applied by H. Nakano [6] in the classical case of uniform spaces (see Theorem 3.1) The saturation of Q, with respect to I_Q, yields a (saturated) local uniformity in X, topologically equivalent to Q (Theorem 3.2). The respective quotient local uniformity on X/I_Q endows the latter space on it the same topology as that one defined on it by the topology of Q on X (Corollary 3.1). Finally, in Section 4 we consider a locally uniform space $X[Q]$ and the equivalence relation defined in the underlying topological space X by a group of homeomorphisms of X. So we give by Theorem 4.1 a characterization of the case that the image of Q in the corresponding quotient (orbit) space of X be the quotient local uniformity.

I wish to express my heart-left thanks to Professor A. Mallios for many stimulating and enlightening discussions, we have had during the writing of this paper. My thanks are also due to the referee for careful reading of the paper and his remarks.

1. Quotient Local Uniformities

By a *local uniformity* on a set X we mean a filter Q of $X \times X$, such that:

1) Every $V \in Q$ contains the diagonal $\Delta \subseteq X \times X$.
2) For every $(x, V) \in X \times Q$, there exists $W \in Q$ such that $(W \circ W)[x] \equiv W^2[x] \subseteq V[x]$.
3) Q is symmetric; i.e., $\overset{-1}{V} \in Q$ for every $V \in Q$.

A pair (X, Q) with Q a local uniformity on X is called a *locally uniform space* and is denoted by $X[Q]$. A map $f : X[Q] \to Y[S]$ between two locally uniform spaces is called *locally uniformly continuous* if $\overset{-1}{\widehat{f \times f}}(S) \subseteq Q$. On the other hand, for any map $f : X[Q] \to Y$, one can define, by a standard argument, the *largest local uniformity* on Y making f locally uniformly continuous (cf., for instance, [9: p. 81, Theorem 3.1] for the case of semi-uniformities).

Now, if R is an equivalence relation on a locally uniform space $X[Q]$ and $p : X \to X/R$ the (canonical) quotient map, the largest local uniformity on X/R making p locally uniformly continuous is called *(the) quotient local uniformity* on X/R. On the other hand, Q is said to be saturated for R, if there exists a basis $\mathcal{B}$ of Q, such that

$$V^S \equiv R \circ V \circ R = V, \tag{1.1}$$

for every $V \in \mathcal{B}$.

Furthermore, we consider the following families of sets:

$$\dot{Q} \equiv \{(p \times p)(V) \equiv \dot{V} : V \in Q\} \tag{1.2}$$

$$Q^S = \{V^S : V \in Q\} \tag{1.3}$$

(see (1.1)). *These are* in fact *filter bases*. Yet, we set

$$D = \{M \subseteq X/R \times X/R : \overset{-1}{\widehat{p \times p}}(M) \in Q\}. \tag{1.4}$$

Thus, we first have the following result, relating the previous three families of sets.

Proposition 1.1 *Let $X[Q]$ be a locally uniform space and R an equivalence relation on X. Then, one has*

$$\dot{Q} = \dot{\widehat{Q^s}} = D. \tag{1.5}$$

Proof. We first note that, for any $V \in Q$ and $(x, y) \in X \times X$, one concludes that (see also (1.1))

$$(\dot{x}, \dot{y}) \in \dot{V} \ \text{ if, and only if, } \ (x, y) \in V^S, \tag{1.6}$$

where we set $\dot{x} = p(x)$, $x \in X$.

Now, we first prove that $D = \dot{Q}$: Indeed, if $M \in D$, then (cf. (1.4)) $\overset{-1}{\widehat{p \times p}}(M) \in Q$ and hence (see also (1.2)), $M = (p \times p)(\overset{-1}{\widehat{p \times p}})(M) \in \dot{Q}$; On the other hand, since $V \subseteq \overset{-1}{\widehat{p \times p}}((p \times p)(V)) \in Q$, for every $V \in Q$, one finally gets the desired relation. Furthermore, as follows from (1.6),

$$(p \times p)(V^S) = \dot{V}, \tag{1.7}$$

for every $V \in Q$, which proves that $\dot{\widehat{Q^S}} = \dot{Q}$ (see also (1.2)), and this terminates the proof. □

In this respect, by virtue of the property 2) of the local uniformity Q, as above, we conclude that:

For every $(x, U) \in x \times Q$, there exists $V \in Q$ such that

$$(V^S \circ V^S)[x] \subseteq U^S[x]. \tag{1.8}$$

Equivalently (see (1.6)), one gets

$$\dot{V} \circ \dot{V}[\dot{x}] \subseteq \dot{U}[\dot{x}]. \tag{1.9}$$

Furthermore, we still have

$$\overset{-1}{\widehat{V^S}} = (\overset{-1}{V})^S \ \text{ and } \ \overset{-1}{\widehat{\dot{V}}} = \dot{\widehat{\overset{-1}{V}}}, \tag{1.10}$$

for every $V \in Q$

Thus, based on the last two relations, as well as, on the equivalent ones (1.8) and (1.9), we now get the following lemma, concerning the filter bases $\dot{Q}$ and Q^S.

Lemma 1.1 *Let $X[Q]$ be a locally uniform space endowed with an equivalent relation R. Then, $\dot{Q}$ is a base of a local uniformity if, and only if, this is the case for Q^S.*

We summarize the preceding, together with a crucial remark on D, into the form of the following.

Theorem 1.1 *Let $X[Q]$ be a locally uniform space and R an equivalence relation on X. Then, the following assertions are equivalent:*

1) *The family D (cf. (1.4)) is a base of the quotient local uniformity on X/R.*

2) *The relation (1.8) (hence, equivalently, (1.9)) holds.*

3) *Any one of the following equal families (filter bases) $\dot{Q}$, $\widehat{\dot{Q}^S}$, D is a base of a local uniformity.*

Proof. As follows from Lemma 1.1, Proposition 1.1 and the equivalent relations (1.8), (1.9), assertions 2) and 3) are equivalent.

On the other hand, we remark that if 3) *is valid, then $\dot{Q}$, hence D as well* (Proposition 1.1), *is a base of the quotient local uniformity* on X/R; Indeed, if Q' is a local uniformity on X/R making p locally uniformly continuous, then $\overset{-1}{\widehat{p \times p}}(Q') \equiv Q$. Therefore, one gets (see also (1.2))

$$Q' = (p \times p)\overset{-1}{\widehat{p \times p}}(Q') \subseteq (p \times p)(Q) = \dot{Q}. \tag{1.11}$$

Thus, 3)⇒1). Of course 1)⇒3) (see also Proposition 1.1), and this completes the proof. □

Remarks 1.1 *i) Given a locally uniform space $X[Q]$, Q is saturated (see (1.1)) if, and only if,*

$$Q^S = Q. \tag{1.12}$$

But then (cf. Theorem 1.1), D is a base of the quotient local uniformity.

ii) As a consequence of (1.6), one gets (cf. also (1.1));

$$V^S := R \circ V \circ R = \overset{-1}{\widehat{(p \times p)}}(p \times p)(V), \tag{1.13}$$

for every $V \in Q$. Accordingly, one has the relation

$$Q^S = \overset{-1}{\widehat{(p \times p)}}(D) \tag{1.14}$$

(see also (1.3), (1.4)).

iii) As follows from the proof of Theorem 1.1, if D is a base of a local uniformity on X/R, then this is, in effect, the quotient local uniformity. (In this regard, cf. also (1.5)).

Scholium 1.1 *Concerning the above Theorem 1.1 and, in conjunction with (1.8), one gets, as a consequence of the technique applied, the following strengthening of a result in [14; p. 69, Lemma 4.10] for quotient uniformities. Thus, we have the next.*

Theorem 1.2 *Let $X[Q]$ be a uniform space and R an equivalence relation on X. Then the following assertions are equivalent:*

1) *The family $\dot{Q}$ (cf. (1.2)) is a basis for the quotient uniformity on X/R.*

2) *For every $U \in Q$ there exists $V \in Q$ such that (see also (1.1)).*

$$V^S \circ V^S \subseteq U^S. \tag{1.15}$$

Proof. Based on [14: p. 69, Lemma 4.10, (i: (a))], it suffices to prove that:

For every $U \in Q$ there exists $V \in Q$ with

$$V \circ R \circ V \subseteq R \circ U \circ R \equiv U^S. \tag{1.16}$$

Thus, we have to prove that (1.15) *and* (1.16) *are equivalent*: Indeed, if (1.16) is valid, one gets

$$R \circ V \circ R \circ R \circ V \circ R \subseteq R \circ R \circ U \circ R \circ R$$

so that, since $R \circ R = R$, one obtains

$$(R \circ V \circ R) \circ (R \circ V \circ R) \subseteq R \circ U \circ R,$$

that, is, (1.15). Conversely, since $V \subseteq V^S \subseteq R \circ V \circ R$, one has $V \circ R \circ V \subseteq V^S \circ R \circ V^S$, that is $V \circ R \circ V \subseteq V^S \circ V^S$ and this terminates the proof. □

Furthermore, based on Lemma 1.1, one can still prove that *condition* 1) *of the previous Theorem is equivalent with the hypothesis that* Q^S (cf. (1.3)) *is the basis of the quotient uniformity on* X/R.

2. Quotient Topologies

We first have the following result.

Theorem 2.1 *Let $X[Q]$ be a locally uniform space and R an equivalence relation in X. Moreover, assume that $\dot{Q}$ (cf. (1.2)) is a (base of a) local uniformity on X/R. Then, one has*

$$\mathcal{T}_{\dot{Q}} = \mathcal{T}^q_{Q^S}. \tag{2.1}$$

(The two members of (2.1) stand for the topology on X/R induced by $\dot{Q}$, and the quotient topology of X/R defined by $\mathcal{T}_{Q^S}$, is the topology of Q^S on X, respectively).

Proof. We first note that by hypothesis for $\dot{Q}$ and Lemma 1.1, $\mathcal{T}_{\dot{Q}}$ and $\mathcal{T}_{Q^S}$ have the above ascribed meaning. In this respect, we recall that one has, by definition,

$$\mathcal{T}_{Q^S} = \{A \subseteq X : \forall x \in A \exists V \in Q,\ V^S[x] \subseteq A\} \tag{2.2}$$

Accordingly, if $M \in \mathcal{T}^q_{Q^S}$, $p^{-1}(M) \in \mathcal{T}_{Q^S}$, so that by (2.2) one gets

$$p^{-1}(M) = \bigsqcup_x V^S_X[x], \tag{2.3}$$

such that $V_X \in Q$, with $x \in p^{-1}(M)$. Therefore, one has (see also (1.6));

$$M = p(p^{-1}(M)) = \bigsqcup_x \widehat{\dot{V}^S_X}[\dot{x}], \tag{2.4}$$

so that (cf. also (2.2)), $M \in \mathcal{T}_{\dot{Q}}$ according to (1.5).

So it remains to prove that

$$p : X[\mathcal{T}_{Q^S}] \to (X/R)[\mathcal{T}_{\dot{Q}}] \tag{2.5}$$

is continuous: Indeed, since

$$V^S = \overset{-1}{\widehat{(p \times p)}}(\dot{V}), \tag{2.6}$$

for every $V \in Q$ (cf. (1.13)), if follows that p *is locally uniformly continuous;* now, analogously to the case of uniform spaces, one proves, that p is also continuous, and this terminates the proof. □

On the other hand, one has by definition (see (1.3)) that $Q^S \subseteq Q$; hence, if Q *is a base of a local uniformity* (cf. Theorem 1.1), one gets $\mathcal{T}_{Q^S} \leq \mathcal{T}_Q$ (see also the notation in (2.1)). So in that case by further assuming that

$$\mathcal{T}_Q \leq \mathcal{T}_{Q^S} \tag{2.7}$$

(hence, in fact, coincidence of $\mathcal{T}_Q$ and $\mathcal{T}_{QS}$), one gets that (2.7) *is equivalent with the following condition*;

$$\text{for every } (x, V) \in X \times Q, \text{ there exists } W \in Q \text{ such that } W^S[x] \subseteq V[x]. \tag{2.8}$$

Yet, the previous two equivalent conditions are still equivalent with the following one;

$$Q^S \text{ is a base of (the local uniformity) } Q. \tag{2.9}$$

The preceding together with Theorem 2.1 (cf. also Theorem 1.1) provide, in fact, the proof of the following.

Theorem 2.2 *Let $X[Q]$ be a locally uniform space and R an equivalence relation on X. Moreover, suppose that Q^S (or equivalently $\dot{Q}$ (cf. Theorem 1.1)) is a base of a local uniformity. Finally, assume that any one of the above three equivalent conditions ((2.7), (2.8), (2.9)) holds. Then, one gets*

$$\mathcal{T}_{\dot{Q}} = \mathcal{T}^q_Q = \mathcal{T}^q_{Q^S}. \tag{2.10}$$

On the other hand, when Q^S is a base of a local uniformity, then the relation $\mathcal{T}_{Q^S} \leq \mathcal{T}_Q$ implies $\mathcal{T}^q_{Q^S} \leq \mathcal{T}^q_Q$ for the respective quotient topologies in X/R. Now, the condition

$$\mathcal{T}^q_Q \leq \mathcal{T}^q_{Q^S}, \tag{2.11}$$

that is, in effect, the coincidence of $\mathcal{T}^q_Q$, $\mathcal{T}^q_{Q^S}$, is equivalent with

$$\begin{aligned} &\text{for every } (x, V) \in X \times Q, \text{ there exists } W \in Q \text{ such that} \\ &p(W^S[x]) = \widehat{\dot{W}^S}[\dot{x}] \subseteq p(V[x]). \end{aligned} \tag{2.12}$$

(Concerning the equality sign in the last relation above, see also (1.6)).

So the equivalence of (2.11) and (2.12), together with (2.10) and Theorem 1.1, yield now the following

Theorem 2.3 *Let $X[Q]$ be a locally uniform space and R an equivalence relation on X. Moreover, assume that D (cf. (1.4)) is a base of the quotient local uniformity on X/R (see Theorem 1.1). Then, (2.12) (or, equivalently, (2.11)) is a necessary and sufficient condition for the following;*

$$\mathcal{T}_D = \mathcal{T}^q_Q = \mathcal{T}_{\dot{Q}} = \mathcal{T}^q_{Q^S}. \tag{2.13}$$

3. The atom of a local uniformity

By applying the notion of "atom" (see [7] for the case of uniform spaces), we give below one more characterization of the quotient local uniformity, as the latter was employed before. For further applications of the concept of atom in case of generalized uniform space, we also refer to [13].

More precisely, suppose we have a locally uniform space $X[Q]$. Then the atom of Q denoted by I_Q, is by definition the intersection of all the members of Q; i.e., we set

$$I_Q := \bigcap_{V \in Q} V. \tag{3.1}$$

Thus, we first have the following.

Theorem 3.1 *Let $X[Q]$ be a locally uniform space. Then, the atom of Q, I_Q, defines an equivalence relation on X.*

Proof. It is obvious, by the very definitions, that I_Q contains the diagonal $\Delta \subseteq X \times X$ so that I_Q defines a reflexive relation in X. On the other hand, the given local uniformity Q has a basis, say $\mathcal{B}$, of symmetric entourages (cf. [13] and/or [8]); hence, one has

$$I_Q = \bigcap_{V \in Q} V = \bigcap_{B \in \mathcal{B}} B, \tag{3.2}$$

so that *the relation* (atom) $I_Q \subseteq X \times X$ *is* also *symmetric*. It remains to prove that I_Q defines a *transitive relation*, as well: In this respect, we first remark that by taking any power of Q, say Q^n, the resulted local uniformity on X defines the same topology on X as Q itself (see [15] and/or [8], [13]). Therefore, for $n = 2$, one obtains

$$\left(\bigcap_{V \in Q} V^2\right)[x] = \bigcap_V V^2[x] = \bigcap_V V[x] = \left(\bigcap_V V\right)[x], \tag{3.3}$$

for every $x \in X$, so that one has

$$I_Q = \bigcap_V V = \bigcap_V V^2. \tag{3.4}$$

On the other hand, it is easily shown that

$$\left(\bigcap_V V\right)^2 \subseteq \bigcap_V V^2; \tag{3.5}$$

hence, by (3.4), one gets $I_Q^2 \subseteq I_Q$, which proves the assertion concerning the transitivity of I_Q, and the proof is complete. □

Scholium 3.1 *An analogous result to Theorem 3.1 is also obtained in the case of "generalized uniform spaces" [8]. More precisely, if $X[Q]$ is a generalized uniform space, then the atom (see (3.1)) defines a preorder relation on X (cf. [13]). However, if Q has a basis of symmetric encourages, then I_Q yields an equivalence relation. This is, for instance, the case if X is an R_0-space; cf. [8: p. 16] and/or [11: p. 274]. Indeed, one proves that: a generalized uniform space $X[Q]$ (cf. [8], [11]) has the underlying topological space $X\ [\mathcal{T}_Q]$ is an R_0-space if, and only if, I_Q is symmetric (as a subset of $X \times X$). In this regard, see also [6: p. 13, Theorem 1.15], as well as [3: p. 6, Proposition]; the latter constitute thus particular cases of the previous assertion. But, for more details, see also [13].*

As a consequence of Theorem 3.1, one now obtains the following.

Theorem 3.2 *Let $X[Q]$ be a locally uniform space and $\mathcal{B}$ a base of Q. Then, the family (see also (3.1))*

$$\mathcal{B}_{at} := \{B_{at} \equiv I_Q \circ B \circ I_Q : B \in \mathcal{B}\} \tag{3.6}$$

is a base of a saturated local uniformity Q_{at} on X, hence, $Q_{at} = Q^s_{at}$ (see (1.12)). Moreover, one has

$$Q_{at} \subseteq Q \quad \text{and} \quad \mathcal{T}_{Q_{at}} = \mathcal{T}_Q. \tag{3.7}$$

Proof. We first prove that (3.6) *provides a base of a local uniformity of X*; that is, we prove that, for any $x \in X$ and $U \in \mathcal{B}$, there exists $V \in \mathcal{B}$ such that (see (3.6))

$$(V_{at} \circ V_{at})[x] \subseteq U_{at}[x] \tag{3.8}$$

(see also Section 1). Now, we recall that (cf. [12: Theorem 1.1]) Q^3 is also a base of a local uniformity of X, inducing on it the same topology on X as Q. Therefore, for any given $(x, U) \in X \times Q$ there exists $V, W \in Q$ such that

$$(V^3 \circ V^3)[x] \subseteq W^3[x] \subseteq U[x] \subseteq U_{at}[x]. \tag{3.9}$$

But, in virtue of (3.4), one also gets

$$V_{at} := I_Q \circ V \circ I_Q \subseteq V \circ V \circ V \equiv V^3, \tag{3.10}$$

for every $V \in Q$. This, together with (3.9), implies (3.8). Of course, since $V \subseteq V_{at}$, for every $V \in Q$, one concludes the first of (3.7); thus *it remains to prove the second one*:

Indeed, we first remark that the first of the relations implies that $\mathcal{T}_{Q_{at}} \leq \mathcal{T}_Q$. On the other hand, by employing an analogous argument as in (3.9), one concludes that (see also (3.10)) for any $(x, U) \in X \times Q$, there exists $V \in Q$ such that

$$V_{at}[x] \subseteq V^3[x] \subseteq U[x] \tag{3.11}$$

that is, one has $\mathcal{T}_Q \leq \mathcal{T}_{Q_{at}}$, as well, and this terminates the proof. □

Corollary 3.1 *Let $X[Q]$ be a locally uniform space and Q_{at} the local uniformity on X defined by Theorem 3.2. Then, $\dot{Q}_{at}$ (relative to the equivalence relation (3.4), see also (1.2)) is (a base of) the respective quotient local uniformity, compatible with the quotient topology, induced on X/I_Q by $\mathcal{T}_Q$.*

Proof. Since Q_{at} is a local uniformity (Theorem 3.2), one concludes that ($\dot{Q}$, as well as) $\dot{Q}_{at}$ is a base of the corresponding (to I_Q) quotient local uniformity (Theorem 1.1); thus, the second of (3.7), together with Theorem 2.2, yield now the relations (see also 1.5) and (2.10)):

$$\mathcal{T}_Q = \mathcal{T}^q_{\dot{Q}} = \mathcal{T}^q_{Q_{at}} = \mathcal{T}_{\dot{Q}_{at}} \tag{3.12}$$

which prove the assertion. □

4. Local uniformities in orbit spaces

We consider below an application of the preceding (more precisely of Theorem 1.1) to the case that the equivalence relation on a given locally uniform space $X[Q]$ *is defined by a group* G *of homeomorphisms* of the respective topological space $X[\mathcal{T}_Q]$.

Thus, for any $x \in X$, we denote by $\mathcal{O}_x$ the orbit of x under the action of the group G, as above; i.e., we set $\mathcal{O}_x := \{ax : a \in G\}$. For convenience, we write ax instead of $a(x)$, for any given $a \in G < Aut(X)$. Moreover, if $A \subseteq X$, one sets

$$\mathcal{O}(A) = \bigcup_{x \in A} \mathcal{O}_x. \tag{4.1}$$

So one now gets the following.

Theorem 4.1 *Suppose that $X[Q]$ is a locally uniform space and G a group of homeomorphisms of the topological space $X[\mathcal{T}_Q]$. Then, the family $\dot{Q}$ (cf. Proposition 1.1) is a base of the quotient local uniformity if, and only if, the following condition holds: For any $(x, U) \in X \times Q$, there exists $V \in Q$, such that*

$$\mathcal{O}(V[\mathcal{O}(V[\mathcal{O}_x])]) \subseteq \mathcal{Q}(U[\mathcal{O}_x]). \tag{4.2}$$

Proof. Denoting by R the equivalence relation on X defined by the given group G of homeomorphisms, let Q^S be the respective filter base defined by (1.3). So based on Theorem 1.1 and Proposition 1.1, if suffices to prove (1.8). Indeed, we prove that:

$$\mathcal{O}(U[\mathcal{O}_x]) = U^S[x] \tag{4.3}$$

as well as, that

$$\mathcal{O}(V[\mathcal{O}(V[\mathcal{O}_x])]) = (V^S \circ V^S)[x], \tag{4.4}$$

for every $x \in X$:

Thus, $y \in \mathcal{O}(U[\mathcal{O}_x])$, iff there exists $\alpha, \beta \in G$ and $x', z \in X$ such that $x' = \beta x$, viz. $(x, x') \in R$, $(x', z) \in U$ and $y = \alpha z$, viz. $(x, y) \in R$; hence, $(x, y) \in R \circ U \circ R \equiv U^S$ (see also (1.1)), i.e., $y \in U^S[x]$, which proves (4.3). Now, an analogous argument can be applied for (4.4), and this terminates the proof. □

References

[1] R.W. Bagley, *Invariant uniformities for coset spaces.* Math. Scand. 14(1964), 19–20.

[2] J.W. Carlson, *Quotient structures for quasi-uniform spaces.* Coll. Math. 36(1976), 63–68.

[3] P. Fletcher-W.F. Lindgren, *Quasi-Uniform Spaces.* Marcel Dekker, New York, 1982.

[4] F. Jeschek, *Compactness in function spaces with a locally uniform range.* Math. Nachr. 85(1978), 267–271.

[5] C.J. Himmelberg, *Quotient uniformities.* Proc. Amer. Math. Soc. 17(1966), 1385–1388.

[6] M.G. Murdershwar and S.A. Naimpally, *Quasi-uniform Topological Spaces.* Noordhoff, Amsterdam, 1966.

[7] H. Nakano, *Uniform Spaces and Transformation Groups.* Wayne State Univ. Press, Detroit, Michigan, 1968.

[8] E. Papatriantafillou, *Spaces with a semi-uniform structure and transformation groups.* Thesis, Univ. of Athens, 1971. Bull. Soc. Math. Grèce 12(1971), 142–222 [Greek]. Math. Rev. 45(1973 # 7668.

[9] E. Papatriantafillou, *Semi-uniformities in quotient spaces.* Bull. Soc. Math. Grèce 12(1971), 79–90.

[10] E. Papatriantafillou, *Invariant semi-uniformities in coset spaces.* Bull. Soc. Math. Grèce 12(1971), 158–164.

[11] E. Papatriantafillou, *Semi-uniformities and semitopological homeomorphism groups.* Rev. Roumania Math. pures appl. 29(1984), 273–276.

[12] E. Papatriantafillou, *On a form of Ascoli's theorem in locally uniform spaces.* Portugal. Math. 46(1989), 385–390.

[13] E. Papatriantafillou, *Generalized Uniform Structures.* (book, to appear).

[14] W. Roelcke-S. Dierolf, *Uniform Structures on Topological Groups and their Quotients.* Mc Graw-Hill, New York, 1981.

[15] J. Williams, *Locally uniform spaces.* Trans. Amer. Math. Soc. 168(1972), 435–469.

◇ Errikos Papatriantafillou
38, Emm. Benaki Str.
10678, Athens, GREECE