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# Linear and non linear body wave inversion

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**Abstract**

In the present study a methodology is developed to estimate the source parameters of an earthquake using teleseismic body wave inversion. The focal mechanism is determined from the second order moment tensor which is related with a double couple source. An initial solution is obtained using the generalized SVD method. Then, by imposing a double couple solution additional constrains are introduced. The minimization between observed and synthetics permits the calculation of the Lagrange multipliers and following, the determination of the components of the seismic moment tensor. Finally, the solution of the nonlinear problem is obtained by applying the Newton-Gauss method using an iterative process.

## 1. Introduction

Seismic waves generated by earthquakes provide information about the source properties and the medium in which the waves propagate. The propagation effects as well as the source effects characterize the structure of the observed seismogram. Mathematically each one of these effects can be calculated in order to generate synthetic seismograms that can directly be compared to the corresponding observed ones. The formalism of comparing synthetic and observed seismograms is known as waveform modeling, a process in which differences between the observed and synthetics seismograms are minimized [1], [7]. In the case where the obtained fitting is acceptable for a certain number of observations, the source parameters of the earthquake can be evaluated.

The seismic moment tensor constitutes the most important source parameter, since it describes in a first order approximation the equivalent forces applied on a fault plane and can be calculated by body wave modeling. Gilbert (1970) introduced a formalism to calculate the displacement at the free surface expressing it as a sum of elementary moment tensors times the corresponding Green's function, where the last represents the response of the medium. This representation is in agreement with physical models where a sudden relative displacement on a fault surface is produced due to a double couple [1].

Seismic moment tensor is represented by a symmetric matrix $3 \times 3$ and can be determined by eigenvalues and eigenvectors analysis. The sum of the eigenvalues describes the isotropic component of the moment tensor and if it is vanished the applied forces constitute a pure double couple source. In this case the seismic moment tensor has only deviatoric components [17], [20]. In general a seismic moment tensor can be decomposed in an isotropic part designating volume changes, a pure double couple indicating slip movement along the fault surface and a compensated linear vector dipole (CLVD) describing seismic sources with no volume changes, net forces or net moment. In case of an earthquake the moment tensor elements can be evaluated by linear inversion using body waves of observed seismograms. As a result the focal mechanism can be determined as well as the scalar seismic moment. By introducing a non uniform slip along the fault surface, the problem becomes non linear and can be resolved using different approximations.

Different methodologies allow the calculation of the source parameters both in time [5],[11], [15], [16], [18], [25], [26], [29], [34], and in frequency domain [3], [4], [33]. A generalized inverse technique based on the moment tensor formalism is proposed [2]. Inversion techniques have also been developed [19], [21], [32] , [12] , [22] in order to determine the source parameters. Kikuchi and Kanamori presented (1982, 1986, and 1991) a series of studies to determine the focal mechanism as well as the rupture pattern of an earthquake which is divided in a sequence of subevents distributed on the fault plane.

Taking into account previous theories both the linear and the non linear problem will be resolved. In the first case the components of the seismic moment tensor will be calculated by inverting observed waveforms. Mathematically, the inverse problem is described by the equation $d = Gm$. The dimension of the data matrix $d$ is $n \times 1$, $G$ is a non square matrix with dimension $n \times m$, composed by a set of five elementary Green's function and the dimension of the model parameters matrix $m$ is $m \times 1$. Generally the system is overdetermined and can be solved using the singular value decomposition method. For the non linear problem the solution is achieved first by minimizing the errors between observed and synthetics [31], [36] and second using the technique of Taylor's series expansions the problem is linearized. [13], [14]. Finally the solution will be obtained using an iterative process.

## 2. Moment tensor representation

### 2.1. *Faulting sources*

An earthquake is a sudden rupture phenomenon that takes place in the interior of the earth caused by tectonic loading. Shearing motions on a fault occur when the elastic strain overcomes the static stress. The rupture propagation generates elastic waves that are recorded in the Earth surface called observed seismograms. The expansion of the rupture area is described by the source time function that can be determined by body wave modeling. Thus, the study of these waves is very important for the determination of the seismic sources. A fault plane can be represented by three parameters $\phi$, $\delta$, $\lambda$ which are the strike, the dip and the rake (figure 1). An arbitrary double couple acts on a fault plane with a slip vector d and a normal to the fault plane $n$. In this case the moment tensor $M_kl$ can be defined using the two vectors **d** and **n** [28]:

$$M_{kl} = \mu A(d_k \mathbf{n}_j + d_j \mathbf{n}_k) \tag{1}$$

where $\mu$ is the shear modulus and $A$ the fault surface. The orientation of the fault plane in geographic coordinates is defined by the two angular parameters the strike $\phi$ and the dip $\delta$. The third parameter $\lambda$ describes the movement of the hanging wall block relative to the footwall block. Taking into account the total average displacement $D$, the scalar seismic moment is equal to $Mo = \mu AD$ [1]. Thus, to characterize the movement on a fault plane the parameters $\phi$, $\delta$ and $\lambda$ must be known. Generally, the components of the moment tensor are calculated using the following relations:

$$M_{11} = -M_0 \left( sin\delta cos\lambda sin2\phi + sin2\delta sin\lambda sin^2\phi \right) \tag{2}$$

$$M_{12} = M_0 \left( sin\delta cos\lambda cos2\phi + \frac{1}{2} sin2\delta sin\lambda sin2\phi \right) \tag{3}$$

$$M_{13} = -M_0 \left( cos\delta cos\lambda cos\phi + cos2\delta sin\lambda sin\phi \right) \tag{4}$$

$$M_{22} = M_0 \left( sin\delta cos\lambda sin2\phi - sin2\delta sin\lambda cos^2\phi \right) \tag{5}$$

$$M_{33} = M_0 \left( sin2\delta sin\lambda \right) = -\left( M_{11} + M_{22} \right) \tag{6}$$

$$M_{23} = -M_0 \left( cos\delta cos\lambda sin\phi - cos2\delta sin\lambda cos\phi \right) \tag{7}$$

$$M_{21} = M_{12} \tag{8}$$

$$M_{31} = M_{13} \tag{9}$$

$$M_{32} = M_{23} \tag{10}$$

The determination of the focal mechanism parameters can be obtained by body wave modeling where the components of the seismic moment tensor are calculated. Following, a methodology to determine the model parameters is presented.

### 2.2. *Displacement field calculation*

The seismic sources can be represented by equivalent forces, producing displacements on the earth's surface, identical to those created during the physical process at the

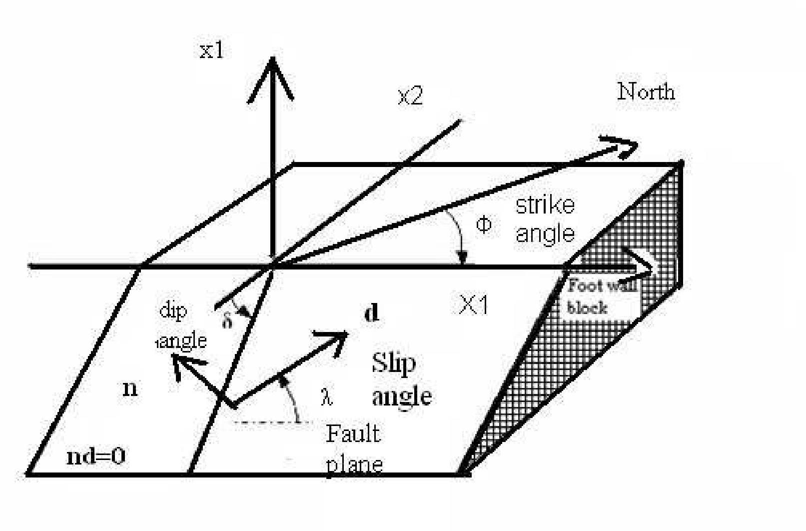


Figure 1: Standard definition of fault plane and slip vector orientation parameters

source. The displacement field $u_n(x,t)$ using the representation theorem for body - waves [1] can be calculated at a position $x$ and time $t$ by the following equation:

$$u_n(x,t) = \int_{-\infty}^{+\infty} d\tau \iint_{\Sigma} [u_i(\xi,\tau)] \cdot c_{ijkl} \cdot \eta_j \cdot \frac{\partial G_{nk}\,(x, t-\tau; \xi, 0)}{\partial \xi_l} d\Sigma \tag{11}$$

where $u_i(\xi,\tau)$ is the definition of dislocation, $G_{nk}$, $l$ is the partial derivative of the Green's function and expresses the nth component of the displacement at position $x$ and time $t$. Note that $x$ is a vector that denotes the position of a station and $\xi$ denotes the position vector of a point source on the rupture surface. $\Sigma$ is the fault plane surface, $\eta_j$ is the $jth$ component of the $n$ which is the vector normal to $\Sigma$. Furthermore, $c_{ijkl}$ is the elastic constant tensor of Hooke law while the term $u_i(\xi,\tau)c_{ijkl}$ denotes the moment density tensor. Taking the derivative with respect to the $\tau$, the following relation is obtained:

$$u_n(x,t) = \iint_{\Sigma} u_i\,(\xi,\tau) \cdot c_{ijkl} \cdot \eta_j * G_{nk,l} \tag{12}$$

By defining the moment density tensor $m_{kl}$ as:

$$m_{kl} = \iint_{\Sigma} [\ u_i\,(\xi,\tau)] \cdot c_{ijkl} \cdot \eta_j \tag{13}$$

the equation (12) is written:

$$u_n\,(x,t) = m_{kl} * G_{nk,l} \tag{14}$$

Integrating the moment density tensor $m_{kl}$ on the surface $\Sigma$ and assuming that all the components of the seismic moment tensor have the same time dependence $s(t)$ the displacement can be written [17]:

$$u_n(x,t) = M_{kl} \cdot [s(t) * G_{nk,l}] \tag{15}$$

where $s(t)$ represents the source time function and $M_{kl}$ are the components of second order seismic moment tensor. This equation denotes the linear relationship between the moment tensor and the Green's functions [9]. In case where the source time function is a delta function, the displacement can be calculated by the next equation:

$$u_n(x,t) = M_{kl} \cdot G_{nk,l} \tag{16}$$

The dimension of the symmetric matrix $M_{kl}$ is $3 \times 3$ and depends of the type faulting. The three diagonal elements represent vector dipoles, while the six off-diagonal elements represent force couples. Considering no volume change, the trace of the matrix is equal to zero. Otherwise there exists an isotropic part, where positive values indicate explosion and negatives implosion. In the case where the determinant is equal to zero, the deviatoric moment tensor represents a pure double couple (DC). In general, the moment tensor can be decomposed in an isotropic part, in a pure double couple and a compensated linear vector dipole (CLVD). The applied forces in a 3D cartesian

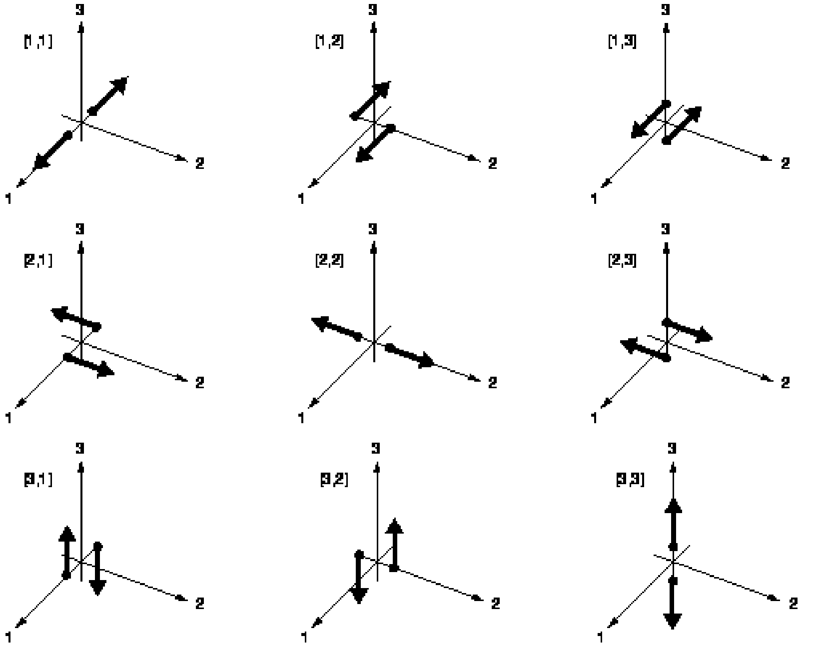


Figure 2: 3D representation of equivalent body forces (Aki and Richards, 1980)

system are shown in figure 2. The off-diagonal elements of the matrix $M_{kl}$ consist of two equal and opposite's at $k$ direction forces f separated by a certain distance d along the $l$ axis.

### 2.3. *Synthetics calculation*

The seismic moment tensor can be calculated either by long period surface waves [16] or by body waves [26] , [17]. The body wave far field displacement for a double couple source with standard fault orientation parameters $\phi$, $\delta$ and $\lambda$ can be calculated by the equations:

$$U^P(r,t) = \frac{1}{4\pi\rho r a^3} R^P \dot{M}\left(t - \frac{r}{a}\right) \tag{17}$$

$$U^{SV}(r,t) = \frac{1}{4\pi\rho r b^3} R^{SV} \dot{M}\left(t - \frac{r}{b}\right) \tag{18}$$

$$U^{SH}(r,t) = \frac{1}{4\pi\rho r b^3} R^{SH} \dot{M}\left(t - \frac{r}{b}\right) \tag{19}$$

Where $U^P$, $U^SV$, $U^SH$ are the vertical, radial and transverse displacement for the $P$, $SV$,$SH$ waves respectively, $\rho$ is the density, $\frac{1}{r}$ is the geometric spreading in a whole space , $a$, $b$ are the velocities of $P$, $S$ waves respectively and $\dot{M}$ is the moment rate. The radiation patterns $R^P$, $R^{SV}$, $R^{SH}$ which is a geometric description of the amplitude and sense of initial motion distributed over the $P$ and $S$ wavefronts and correspond to $P$, $SV$, $SH$ waves respectively can be calculated using the following relations [1]:

$$R^P = A_1 - A_2 + A_3 + A_4 \tag{20}$$

$$R^{SV} = B_1 - B_2 + B_3 - B_4 \tag{21}$$

$$R^{SH} = C_1 + C_2 + C_3 - C_4 \tag{22}$$

where:

$A_1 = cos\lambda sin\delta sin^2 i_h sin2\Phi, A_2 = cos\lambda cos\delta sin2i_h cos\Phi$
$A_3 = sin\lambda sin2\delta\left(cos^2 i_h - sin^2 i_h sin^2\Phi\right), A_4 = sin\lambda cos2\delta cos2i_h sin\Phi$
$B_1 = sin\lambda cos2\delta cos2i_h sin\Phi, B_2 = cos\lambda cos\delta cos2i_h cos\Phi$
$B_3 = \frac{1}{2}cos\lambda sin\delta sin2i_h sin2\Phi, B_4 = \frac{1}{2}sin\lambda sin2\delta sin2i_h\left(1 + sin^2\Phi\right)$
$C_1 = cos\lambda cos\delta cosi_h sin\Phi, C_2 = cos\lambda sin\delta sini_h cos2\Phi$
$C_3 = sin\lambda cos2\delta cosi_h cos\Phi, C_4 = \frac{1}{2}sin\lambda sin2\delta sini_h sin2\Phi$ ,

$(\phi, \delta, \lambda)$ are the strike, the dip and the rake of the considered fault, $\Phi = \phi - \phi_s$, $\phi_s$ is the azimuth of the station and $i_h$ is the incidence angle at the source.

## 3. Moment Tensor Inversion

Following the above mentioned formulation, the data, denoted by a vector $D$, can be represented by a linear combination of a non square matrix $G$ representing elementary Green's functions and a vector $M$ representing the model parameters:

$$D = G \cdot M \tag{23}$$

The dimension of the matrix $G$ is $n \times m$ where $n$ is the number of observations and $m$ the number of fundamental Green functions. The inverse problem consists of inverting the non symmetric matrix $G$ in order to determine the parameters of the model. Several methods exist to calculate the inverse of the matrix $G$ like the normal equations, QR-decomposition, and the singular value decomposition (SVD). In the present study the singular value decomposition method is chosen and can be used either in overdetermined or underdetermined systems. The first step is to define two symmetrical matrices the $U = GG^T$ and the $V = G^T G$ that have the same eigenvalues. The second step is to define the diagonal matrix $\Lambda$ by calculating the positive square root of the non zero eigenvalues, named singular values [10], [27], [30], [35]. Thus, the model parameter $M$ can be calculated by the relation:

$$M = \left(G^T G\right)^{-1} G^T D \tag{24}$$

where using the SVD method:

$$G = U \Lambda V^T \tag{25}$$

The obtained eigenvectors are parallel to the principal stress axes and the norm of the matrix is equal to the seismic moment. In general a moment tensor $M_{kl}$ is symmetric and has 6 independent elements. In case of an earthquake the trace must be equal to zero (no isotropic part) as well as the determinant of the matrix must also be equal to zero. Considering the two mentional constraints the obtained moment tensor consists of 5 independent elements. Kikuchi and Kanamori (1991) defined that the moment tensor can be decomposed in 5 elementary moment tensors:

$$\mathbf{M^1_{kl}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{M^2_{kl}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{M^3_{kl}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{M^4_{kl}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \mathbf{M^5_{kl}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In this case the moment tensor $M_{kl}$ can be calculated by the linear combination of the 5 elementary moment tensors mentioned above [6]:

$$M_{kl} = \sum_{m=1}^{5} a_m \cdot M^m_{kl} \tag{26}$$

and using the coefficient $a_m$ the above relation becomes [24]:

$$\mathbf{M_{kl}} = \begin{pmatrix} a_2 - a_5 + a_6 & a_1 & a_4 \\ a_1 & -a_2 + a_6 & a_3 \\ a_4 & a_3 & a_5 + a_6 \end{pmatrix} \tag{27}$$

In order to determine the moment tensor $M_{kl}$ a methodology is developed to calculate the parameters $a_m$.

## 4. Deviatoric moment tensor determination

In the following $[obs_n(t)]$ denotes the observed seismograms and $[syn_n(t)]$ the calculated synthetics that correspond to the $nth$ station.The model parameters are estimated by minimizing the difference between observed and synthetics using a least square approach:

$$\Delta_1 = \sum_{i=1}^{n} \int \left[obs_i\left(t\right) - syn_i\left(t\right)\right]^2 dt = min \tag{28}$$

Subsequently:

$$\Delta_1 = \sum_{i=1}^{n} \int \left[obs_i\left(t\right)\right]^2 dt + \sum_{i=1}^{n} \int \left[syn_i\left(t\right)\right]^2 dt - 2\sum_{i=1}^{n} \int \left[obs_i\left(t\right) \cdot syn_i\left(t\right)\right] dt \tag{29}$$

The best solution is the one obtained where $\Delta_1$ is minimized. Considering n observed seismograms $[obs_n(t)]$, $nx5$ green's functions are calculated to create the corresponding $n$ synthetics seismograms $[syn_n(t)]$:

$$syn_n\left(t\right) = \sum_k \sum_l M_{kl} * G_{nk,l} \tag{30}$$

and taking to account the 5 elementary moment tensors:

$$syn_n\left(t\right) = \sum_{m=1}^{5} a_m \left(\sum_k \sum_l M_{kl}^m * G_{nk,l}\right) \tag{31}$$

Let us:

$$S_n^m\left(t\right) = \sum_k \sum_l M_{kl}^m \cdot \left[s\left(t\right) * G_{nk,l}\right] \tag{32}$$

Then the equation (30) becomes:

$$syn_n\left(t\right) = \sum_{m=1}^{5} a_m S_n^m\left(t\right) \tag{33}$$

The equation (28) using the last equation can be written:

$$\Delta_1 = \left[obs_n\left(t\right) - a_m S_n^m\left(t\right)\right]^2 \tag{34}$$

In this case the model parameters can be calculated by minimizing the difference between observed and synthetics using the relation (28). Taking into account the above relation the coefficient $a_m$ can be calculated from the formula:

$$a_m = \left[\left(S_n^m(t)\right)^T S_n^m(t)\right]^{-1} \left[S_n^m(t)\right]^T obs_n(t) \tag{35}$$

or in a more simple equation:

$$a_m^0 = \left(S^T S\right)^{-1} S^T \cdot obs_n(t) \tag{36}$$

where:

$$S^T S = \sum_n \int \left[syn_{nm}(t) \cdot syn_n(t)\right] dt \tag{37}$$

The equation (36) represents the general solution from which the deviatoric moment tensor can be calculated using the equation (27). Note that this general solution is used as an initial solution for the double couple solution which is described following.

## 5. Double couple solution

In the case of an earthquake the moment tensor must fulfil two constrains concerning the trace and the determinant of the matrix:

$$Trace\left[M_{kl}\right] = 0 \tag{38}$$

$$Det\left[M_{kl}\right] = 0 \tag{39}$$

The first constrain indicates a non isotropic part and the second a double couple source mechanism. The general solution given by (35) doesn't include these conditions. Thus a new set of parameters am must be defined in case of an earthquake. To minimize the equation (34) the method of Lagrange multipliers is applied by defining:

$$\Delta_2 = \Delta_1 + 2\lambda D \tag{40}$$

where $D$ is the determinant of matrix $M_{kl}$ and  the Lagrange multipliers. To solve this equation the partial derivatives of $\Delta_2$ with respect to $a_m$ and  must be taken in consideration:

$$\Delta_1 + 2\lambda D = 0 \tag{41}$$

$$\frac{\partial \Delta_2}{\partial a_m} = 0 \tag{42}$$

$$\frac{\partial \Delta_2}{\partial \lambda} = 0 \tag{43}$$

and using the equation (34) the next equation can be defined:

$$\left[obs_n(t) - a_m \cdot S(t)\right]^2 + 2\lambda D = 0 \tag{44}$$

which becomes:

$$S^T(t)\,S(t)\cdot a_m - S^T(t)\cdot obs_n(t) + \lambda \frac{\partial D}{\partial a_m} = 0 \tag{45}$$

Finally the coefficients $a_m$ can be calculated by the next equation:

$$a_m = \left[S^T(t)\cdot S(t)\right]^{-1}\cdot S^T(t)\cdot obs_n(t) - \lambda\cdot\left[S^T(t)\cdot S(t)\right]^{-1}\frac{\partial D}{\partial a_m} \tag{46}$$

which is equivalent to:

$$a_m = a_m^0 - \lambda\left(S^T S\right)^{-1}\frac{\partial D}{\partial a_m} \tag{47}$$

where the Lagrange multipliers and the partial derivatives of the determinant of the matrix $M_{kl}$ with respect to the elements $a_m$ must calculated. This equation represents the final solution in case of an earthquake, where the moment tensor can be calculated taking into account the new coefficients $a_m$.

The Lagrange multipliers can be determined using the equation:

$$\lambda = -\frac{1}{2}\sum_{m=1}^{5}\frac{\frac{\partial \Delta_1}{\partial a_m}}{\frac{\partial D}{\partial a_m}} \tag{48}$$

The numerator of (48) is equal to:

$$\frac{\partial \Delta_1}{\partial a_m} = \sum_{i=1}^{n}\int\left[syn_n(t)\,obs_n(t)\right]dt \tag{49}$$

and the denominator is the partial derivative of the following expression with respect to the elements $a_m$:

$$D = -a_2^2 a_5 + a_2 a_5^2 + 2a_1 a_3 a_4 + a_2 a_4^2 - a_5 a_1^2 - a_2 a_3^2 + a_3^2 a_5 \tag{50}$$

The last equation is a non linear equation. Different methods exist to solve such problems like the Newton, Newton-Gauss and Levenberg-Marquardt methods. In this study the Newton-Gauss method is used. According to this method, first an initial model is defined; second using the Taylor's technique, the problem becomes linear and finally it is solved by applying the methodology developed in the previous section.

Taking into account a small perturbation $\Delta a_m$ around the initial model $a_m^0$ ($a^0$ is a zero approximation), expressed by a Taylor expansion series:

$$a_m = a_m^0 + \Delta a_m \tag{51}$$

and

$$D = D^0 + \Delta D \tag{52}$$

Then:

$$D\left(a_m\right) = 0 \Leftrightarrow D^0 + \frac{\partial D}{\partial a_m}|_{a^0}\Delta a_m = 0 \tag{53}$$

Applying the relation (51) in the (45) is taken:

$$\left(S^T S\right) \cdot \left(a_m^0 + \Delta a_m\right) - S^T \cdot obs_n\left(t\right) + \lambda \frac{\partial D}{\partial a_m}|_{a^0} = 0 \tag{54}$$

$$S^T S \cdot a_m^0 + S^T S \cdot \Delta a_m - S^T \cdot obs\left(t\right) + \lambda \frac{\partial D}{\partial a_m}|_{a^0} = 0 \tag{55}$$

and by imposing:

$$R_{nm} = S_{nm}^T \cdot S_{nm} \tag{56}$$

$$Q_m = S^T \cdot obs_n\left(t\right) - Ra_m^0 \tag{57}$$

the next set of equation must be resolved:

$$\begin{bmatrix} \sum_m R_{nm} & \left(\frac{\partial D}{\partial a_m}\right)^T \\ \frac{\partial D}{\partial a_m} & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta a_m \\ \lambda \end{bmatrix} = \begin{bmatrix} Q \\ -D^0 \end{bmatrix} \tag{58}$$

By resolving the system (53) the $\lambda$ parameter can be calculated, while using the equation (47) the new coefficients $a_m$ will be determined. Finally, the determination of the model parameters can be obtained by an iteration process.

## 6. Conclusions

Body wave inversion in teleseismic distances is a powerful tool to determine the source parameters of an earthquake and also helps us to understand the physics of the earthquake process. For this purpose Green's functions are generated including wave-propagation effects due to the structure. For the inversion P, SH and SV waves can be used. In several cases SV waves present complexities because they are more sensitive to the structure and for this reason only P and SH waves are used commonly. One of the purposes of this study is to use only a few phases to estimate the source parameters.

The overdetermined problem generally results to a non double couple solution presenting a superposition of two different couples acting at the same point in time and space. By imposing additional constrains, the general solution is separated in a double couple and CLVD parts. In this case the components of the seismic moment tensor can be calculated. The implemented time domain weighted least-square inversion calculates eigenvalues and eigenvectors using the SVD method, where the eigenvectors are the principal axes of the seismic moment tensor. The obtained general solution is used as an initial solution for the non linear case. Following, using the Gauss-Newton method, the problem is linearized and the model parameters can be determined by an iterative process. This procedure can be applied routinely to seismic data to recover and catalogue the source parameters of an earthquake.

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